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THE EIGENFUNCTION EXPANSION METHOD IN DYNAMIC ELECTROELASTICITY PROBLEMS*

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A general formulation is proposed for the eigenfunction expansion method (EEM) in non-stationary dynamic problems of electroelasticity. The ortogonality of the displacement eigenfunctions (EF) in the bulk of the body is proved. It is established that the solution for the displacements has the same form as in classical elasticity theory while the electric field potential contains, in addition to a series in non-orthogonal EF, an unrelated component determined from the solution of the Laplace equation for an anisotropic medium under mixed boundary conditions. The problem of the instantaneous electrical loading of a piezoceramic rod is examined as an example. The specific singularities reflected by the general method are analysed for electroelastic wave fields as compared with elastic fields.

The comparatively rare use of the EEM /1, 2/ as compared with the integral transform method, say, is due to the complexity of seeking eigenfunctions in problems with mixed boundary conditions for bodies bounded by surfaces belonging to different coordinate families or with boundaries going to infinity. However, the solution can be obtained in the latter case by a passage to the limit from a finite to an infinite body /3/. Meanwhile the EEM also has a number of advantages over the integral transform method, among which is the physical clarity of the solution /3/. Certain general properties of the non-stationary wave field in semi-infinite elastic waveguides /4/ can be established successfully by using this method.

1. The complete system of equations of motion of a piezoelectric medium /5, 6/ includes linear equations of state, stress equations of motion, electrostatics equations, and Cauchy relationships

$$\tau_{ij}=c_{ijkl}^{E}\ \varepsilon_{kl}-e_{kij}E_k,\qquad D_i=e_{ikl}\varepsilon_{kl}+\varepsilon_{ik}^{S}E_k$$
$$\tau_{ij,\,j}+F_i-\rho u_i^{\cdot\cdot}=0;\quad D_{i,\,i}=0,\quad E_k=-\varphi_{,\,k};\qquad \varepsilon_{kl}={}^1/_2\,(u_{k,l}+u_{l,k}) \tag{1.1}$$

In these equations τ_{ij} is the mechanical stress tensor ε_{kl} is the strain tensor, E_i is the electric field strength, D_i is the electric induction vector, u_i is the displacement vector, ψ is the electric potential c_{ijkl}^{E} is the elastic constants tensor, measured in a constant electrical field, e_{ikl} is the piezomoduli tensor, ε_{ik}^{S} is the dielectric permittivity tensor for constant strains, F_i is the bulk force vector, and ρ is the material density. The properties of the material constants tensors are described in /5/. The subscript after the comma denotes differentiation with respect to the space coordinate. The Latin subscripts i, j, k, l run through the values 1, 2, 3 and the summation is over repeated subscripts.

Eliminating all variables except u_i and ψ from (1.1), we obtain a system of second-order partial differential equations

$$c_{ijkl}^{E}u_{k,\,lj}+e_{kij}\psi_{,kj}+F_i-\rho u_i^{\cdot\cdot}=0$$
$$e_{ikl}u_{k,li}-\varepsilon_{ik}^{S}\psi_{,ki}=0 \tag{1.2}$$

We examine the following initial-boundary value problem: it is required to find the solution of system (1.2) in a volume V with boundaries $S=S_1+S_2=S_3+S_4$ at each point of which one of the mechanical boundary conditions

$$u_i=f_i\,(x,t),\quad x\in S_1$$
$$n_j\tau_{ij}\equiv n_j\,(c_{ijkl}^{E}u_{k,l}+e_{kij}\psi_{,k})=g_i\,(x,t),\quad x\in S_2 \tag{1.3}$$

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and the electrical boundary conditions /5, 6/

$$\psi = \psi_0 (x, t), \quad x \in S_3 \qquad (1.4)$$
$$n_l D_l \equiv n_l (e_{lkl} u_{k,l} - \varepsilon^S_{ik} \psi_{,k}) = \sigma (x, t), \quad x \in S_4$$

is given.

The f_i on the right-hand sides of conditions (1.3) and (1.4) are given displacements, g_i are external forces vectors, ψ_0 is the given electric potential, σ is the known free charge density, and $x = \{x_i\}$ is the three-dimensional coordinate of the point. We note that, as a rule, the physically realizable electrical boundary conditions have a more particular nature, namely: $\psi_0 (x, t) = \psi_\nu (t)$ $(\nu = 1, 2, \ldots, N)$ on each of the N electrodes comprising the surface S_3 while $\sigma = 0$ on the non-electroded surface (S_4). The general form of notation is used in the theoretical analysis because of its compactness.

The solution of problem (1.2)-(1.4) is defined uniquely if initial conditions for the mechanical variables /6/

$$u_i (x, 0) = u_{i0} (x), \quad u_i^{\cdot} (x, 0) = v_{i0} (x), \quad x \in V \qquad (1.5)$$

are given in addition to the boundary conditions, where u_{i0}, v_{i0} are known functions of the coordinates.

Starting from the general idea of the EEM, we will examine the followng eigenvalue problem: it is required to find the values of the parameter Ω^2, for which the homogeneous boundary-value problem corresponding to (1.2)-(1.4)

$$c^E_{ijkl} U_{k,lj} + e_{kij} \Psi_{,kj} + \rho \Omega^2 U_i = 0 \qquad (1.6)$$
$$e_{ikl} U_{k,li} - \varepsilon_{ik}{}^S \Psi_{,ki} = 0$$

$$U_i = 0, \quad x \in S_1; \quad n_j (c^E_{ijkl} U_{k,l} + e_{kij} \Psi_{,k}) = 0, \quad x \in S_2 \qquad (1.7)$$
$$\Psi = 0, \quad x \in S_3; \quad n_i (e_{ikl} U_{k,l} - \varepsilon_{ik}{}^S \Psi_{,k}) = 0, \quad x \in S_4$$

has the non-zero solutions $U_i (x), \Psi (x)$. The electroelastic field $U_i (x) e^{-i\Omega t}$, $\Psi (x) e^{-i\Omega t}$ corresponds to free vibrations of the volume V under homogeneous mechanical and electrical boundary conditions.

Assuming that there is an infinite set of eigenvalues $\Omega_m{}^2$ $(m = 1, 2, \ldots)$, forming a discrete series (for a bounded volume), we denote the eigenfunctions found by $U_i^{(m)} (x), \Psi^{(m)} (x)$. As in ordinary elasticity theory, the existence of an infinite series $\Omega_m{}^2$ required verification in each specific case, i.e., substantially the solution of problem (1.6) and (1.7).

The proof that the eigennumbers Ω^2 are real and positive largely repeats the reasoning carried out in /2/ for the elastic case. They are based on the use of Gauss's theorem and the condition for the internal energy density of a piezoelectric medium $W = {}^1/_2 (\tau_{ij} \varepsilon_{ij} + E_k D_k)$ to be positive.

To set up the orthogonality of the EF corresponding to different eigenfrequencies Ω_m and Ω_n, we write the identity

$$(c^E_{ijkl} U^{(m)}_{k,lj} + e_{kij} \Psi^{(m)}_{,kj}) U_i^{(n)} - (c^E_{ijkl} U^{(n)}_{k,lj} + e_{kij} \Psi^{(n)}_{,kj}) U_i^{(m)} +$$
$$(e_{ikl} U^{(m)}_{k,li} - \varepsilon^S_{ik} \Psi^{(m)}_{,ki}) \Psi^{(n)} - (e_{ikl} U^{(n)}_{k,li} - \varepsilon^S_{ik} \Psi^{(n)}_{,ki}) \Psi^{(m)} +$$
$$\rho (\Omega_m{}^2 - \Omega_n{}^2) U_i^{(m)} U_i^{(n)} = 0$$

that obviously follows from (1.6).

Integrating over V and using Gauss's theorem and the boundary conditions (1.7), we have

$$(\Omega_m{}^2 - \Omega_n{}^2) \int_V \rho U_i^{(m)} U_i^{(n)} dV = 0 \qquad (1.8)$$

i.e., the vector displacements EF are orthogonal with weight $\rho (x)$ in the bulk of the body. The scalar EF of the electric potential $\Psi^{(m)}$ do not occur in the orthogonality relationship (1.8) and are not generally orthogonal.

We will henceforth assume the displacement EF to be orthonormalized

$$\int_V \rho U_i^{(m)} U_i^{(n)} dV = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases} \qquad (1.9)$$

thereby eliminating the arbitrary multiplies in both $U_i^{(m)}$ and in $\Psi^{(m)}$. We note that the completeness of $U_i^{(m)}$ in V assumed later, just like the structure of the spectrum, will require special study in specific cases.

Having the EF set $U_i^{(m)}$, $\Psi^{(m)}$, we will seek the solution of the initial-boundary value problem (1.2)-(1.5) in the form (the summation is between $m = 1$ and $m = \infty$ everywhere below)

$$u_i = u_i^s(x,t) + \Sigma U_i^{(m)}(x)\, p_m(t), \quad \psi = \psi^s(x,t) + \Sigma \Psi^{(m)}(x)\, p_m(t) \tag{1.10}$$

where u_i^s, ψ^s is the solution of the "static" problem corresponding to problem (1.2)-(1.4) for $u_i^{\cdot\cdot} \equiv 0$ while $p_m(t)$ are as yet unknown functions of time.

The time enters as a parameter in the latter problem, which makes it much simpler compared with the original problem. Moreover, it will later be seen that a knowledge of explicit expressions for u_i^s, ψ^s is not necessary since they drop out of the final solution.

The representations (1.10) satisfy the second of Eqs.(1.2) and the boundary conditions (1.3) and (1.4). Substituting (1.10) into the first equation of (1.2), we obtain, by virtue of the first equation in (1.6) and the corresponding equation in the "static" problem,

$$\Sigma \rho U_i^{(m)} (p_m^{\cdot\cdot} + \Omega_m^2 p_m) = -\rho u_i^{s\cdot\cdot}$$

Multiplying by $U_i^{(n)}$ and integrating over the volume V, taking (1.9) into account, we find

$$p_m^{\cdot\cdot} + \Omega_m^2 p_m = Q_m^{\cdot\cdot}(t), \quad m = 1, 2, \ldots \tag{1.11}$$

$$Q_m(t) = -\int_V \rho u_i^s(x,t)\, U_i^{(m)}(x)\, dV \tag{1.12}$$

The general solution of (1.11) has the form

$$q_m(t) = q_m(0)\cos\Omega_m t + q_m^{\cdot}(0)\frac{\sin\Omega_m t}{\Omega_m} - \Omega_m \int_0^t Q_m(\tau)\sin\Omega_m(t-\tau)\, d\tau \tag{1.13}$$

$$q_m(t) = p_m(t) - Q_m(t)$$

Substituting the first equation of (1.10) into the initial conditions (1.5), we have

$$u_i^s(x,0) + \Sigma U_i^{(m)}(x)\, p_m(0) = u_{i0}(x)$$

$$u_i^{s\cdot}(x,0) + \Sigma U_i^{(m)}(x)\, p_m^{\cdot}(0) = v_{i0}(x)$$

Multiplication by $\rho U_i^{(n)}(x)$ and integration over V yields

$$q_m(0) = \int_V \rho u_{i0}(x)\, U_i^{(m)}(x)\, dV, \quad q_m^{\cdot}(0) = \int_V \rho v_{i0}(x)\, U_i^{(m)}(x)\, dV \tag{1.14}$$

The functions of time $q_m(t)$ satisfy the equations

$$q_m^{\cdot\cdot} + \Omega_m^2 q_m = -\Omega_m^2 Q_m(t) \tag{1.15}$$

because of (1.11).

The initial conditions for $q_m(t)$ are determined by (1.14) and their explicit expressions follow from (1.13).

If it is taken into account that in conformity with (1.12) the series with coefficients $Q_m(t)$ is the expansion of the vector function $-u_i^s(x,t)$ in the complete system $U_i^{(m)}(x)$, the first expansion in (1.10) is written in the form

$$u_i = \Sigma U_i^{(m)}(x)\, q_m(t) \tag{1.16}$$

$$q_m(t) = q_m(0)\cos\Omega_m t + q_m^{\cdot}(0)\frac{\sin\Omega_m t}{\Omega_m} + \frac{1}{\Omega_m}\int_0^t \Phi_m(\tau)\sin\Omega_m(t-\tau)\, d\tau \tag{1.17}$$

The functions $\Phi_m(t)$ introduced here are defined in terms of $Q_m(t)$ for which a representation is successfully obtained by means of simple but awkward calculations analogous to those performed earlier /2/ without using the "static" solution

$$\Phi_m(t) = \int_V F_i U_i^{(m)}\, dV - \int_{S_1} n_j (c_{ijkl}^E U_{k,l}^{(m)} + e_{kij}\Psi_{,k}^{(m)}) f_i(x,t)\, dS + \tag{1.18}$$

$$\int_{S_2} U_i^{(m)} g_i(x,t)\, dS - \int_{S_3} n_i (e_{ikl} U_{k,l}^{(m)} - \varepsilon_{ik}^S \Psi_{,k}^{(m)})\, \psi_0(x,t)\, dS +$$

$$\int_{S_4} \Psi^{(m)} \sigma(x, t)\, dS = -\Omega_m^2 Q_m(t)$$

It follows from expansion (1.10) rewritten taking the functions $q_m(t)$ into account for the potential that here the "static" solution ψ^s is not cancelled by the sum with coefficients $Q_m(t)$ in the general case.

If, as holds in the majority of applied problems, the asymptotic estimate

$$\Omega_m \sim \text{const}\cdot m, \quad m \to \infty \tag{1.19}$$

is valid for the eigenfrequencies Ω_m then the necessity to find ψ^s can be avoided.

Let us represent ψ in the form

$$\psi = \varphi(x, t) + \Sigma \Psi^{(m)}(x)\, q_m(t) \tag{1.20}$$

$$\varphi = \psi^s(x, t) + \Sigma \Psi^{(m)}(x)\, Q_m(t) = \psi^s(x, t) - \Sigma \Omega_m^{-2} \Psi^{(m)}(x)\, \Phi_m(t) \tag{1.21}$$

and let us try to formulate an individual boundary-value problem for the new function φ.

Substitution of the expansions (1.16) and (1.20) into the second equality of (1.2) yields the equation

$$\varepsilon_{ik}^S \varphi_{,ki} = 0, \quad x \in V \tag{1.22}$$

To obtain the boundary conditions we note that by virtue of the properties of the functions $u_i^s, \psi^s, U_i^{(m)}, \Psi^{(m)}$ in the representations (1.10) and the combinations of their derivatives in (1.3) and (1.4) the limits of the sums equal the sums of the limits on appropriate parts of the boundary S. Comparing expressions (1.17) for q_m and the coefficients of the series in (1.21) we see that the latter decrease as $m \to \infty$ not more slowly than $p_m(t)$, and consequently, a term-by-term passage to the limit is also possible in the series obtained by substituting (1.16) and (1.20) into (1.3) and (1.4). We hence find that the function φ should satisfy the boundary conditions

$$\psi = \psi_0(x, t), \quad x \in S_3; \quad -n_i \varepsilon_{ik}^S \psi_{,k} = \sigma(x, t), \quad x \in S_4 \tag{1.23}$$

Therefore, the function $\varphi(x, t)$ is determined from the solution of the unrelated problem, namely, it satisfies the Laplace equation for an anisotropic medium (1.22) under the mixed boundary conditions (1.23).

The general scheme of the EEM in dynamic electroelasticity problems can be represented in the following form. After having found the EF system $U_i^{(m)}, \Psi^{(m)}$ from the solution of problem (1.6) and (1.7), the functions $\Phi_m(t)$ are determined from (1.18). Then the functions $q_m(t)$ are calculated from (1.17) and (1.14). The solution of the initial-boundary value problem (1.2)-(1.5) is represented by (1.16) and (1.20), where the boundary-value problem (1.22) and (1.23) should be solved to determine the unrelated potential $\varphi(x, t)$.

Comparing the scheme obtained with that which occurs in ordinary elasticity theory /2/, the deduction can be made that the "static" displacement u_i^s drops completely out of the solution (1.16) in both cases. Part of the "static" solution still remains in expression (1.20) for the potential in the form of the function φ although the latter is determined from the solution of the simpler problem (1.22)-(1.23). The reason is that the complete system of electroelasticity Eqs.(1.1) uses a quasistatic approximation of Maxwell's equations, and, therefore, allows an instantaneous change in the electric field in the bulk of the material while displacement perturbations propagate at a finite velocity /6/. The function $\varphi(x, t)$ again corresponds to a change in potential in the bulk not accompanied by changes in the displacement and strain fields.

2. Specific singularities of the electroelastic wave fields as compared with the pure elastic fields and reflected by the general scheme of the eigenfunction method appear most clearly in problems on the instantaneous electrical loading (unloading) of piezoceramic bodies /6-8/ and are illustrated by the following example.

A piezoceramic rod $-h < z < h$ is polarized along the z axis. The one-dimensional equations of the piezoelectric effect have the form /6/

$$\varepsilon_z = s_{33}^E \sigma_z + d_{33} E_z, \quad D_z = \varepsilon_{33}^T E_z + d_{33} \sigma_z \tag{2.1}$$

The complete system of dynamic electroelasticity equations include, in addition to (2.1), the equation of motion, the electrostatics equations, and the Cauchy relationship

$$\frac{\partial \sigma_z}{\partial z} = \rho \frac{\partial^2 u_z}{\partial t^2}; \quad \frac{\partial D_z}{\partial z} = 0, \quad E_z = -\frac{\partial \psi}{\partial z}; \quad \varepsilon_z = \frac{\partial u_z}{\partial z} \tag{2.2}$$

We will consider the problem of the instantaneous electrical loading of the rod by the potential difference $2V_0$. The boundary conditions for a mechanically free rod are

$$\psi = \pm V_0, \quad \sigma_z = 0, \quad z = \pm h \tag{2.3}$$

the initial conditions are zero

$$u_z = 0, \quad u_z^{\cdot} = 0, \quad t = 0 \tag{2.4}$$

The solution of problem (2.1)-(2.4) by the method of Sect.1 has the form

$$\begin{gathered} u_z = -d_{33}(1-k_{33}^2)V_0\Sigma\alpha_m \sin\varkappa_m z(1-\cos\Omega_m t) \\ \psi = V_0 zh^{-1} - k_{33}^2 V_0\Sigma\alpha_m(\sin\varkappa_m z - zh^{-1}\sin\varkappa_m h)(1-\cos\Omega_m t) \\ \alpha_m = 2\{[(\varkappa_m h)^2 - k_{33}^2(1-k_{33}^2)]\sin\varkappa_m h\}^{-1}, \quad k_{33}^2 = d_{33}^2(\varepsilon_{33}^T s_{33}^E)^{-1} \end{gathered} \tag{2.5}$$

where k_{33}^2 is the longitudinal coefficient of electromechanical coupling $\varkappa_m = \Omega_m c^{-1}$ are the roots of the transcendental equation

$$\operatorname{ctg}\varkappa_m h = k_{33}^2(\varkappa_m h)^{-1}$$

and $c = [\rho s_{33}^E(1-k_{33}^2)]^{-1/2}$ is the wave propagation velocity in the rod with open electrodes /6/. The system of eigenfunctions $\sin\varkappa_m z$ is orthogonal in $(-h, h)$ /9/ and the estimate that agrees with (1.19)

$$\Omega_m = (m - 1/2)\pi c h^{-1} + o(1), \quad m \to \infty$$

holds for the eigenfrequencies. The component $V_0 zh^{-1}$ in the expression for ψ is analogous to the function φ of the general theory and is a solution of the unrelated problem

$$\frac{\partial^2\varphi}{\partial z^2} = 0, \quad -h < z < h; \quad \varphi = \pm V_0, \quad z = \pm h$$

In the interests of a more detailed analysis of the structure of expansion (2.5) we will find the solution of the same problem in the domain of the complex Fourier transform with respect to time

$$\begin{gathered} u_z^F(z,\omega) = -d_{33}(1-k_{33}^2)V_0(T\omega^2\Delta)^{-1}i\sin(\omega zc^{-1}) \\ \psi^F(z,\omega) = V_0(\omega\Delta)^{-1}i[zh^{-1}\cos\omega T - k_{33}^2(\omega T)^{-1}\sin(\omega zc^{-1})] \\ \Delta = \cos\omega T - k_{33}^2(\omega T)^{-1}\sin\omega T \end{gathered} \tag{2.6}$$

here

$$(u_z^F, \psi^F) = \int_0^\infty (u_z, \psi)e^{i\omega t}dt$$

is the transform of the desired functions, $T = hc^{-1}$ is the time of elastic wave passage over half the rod length. Calculation of the inversion of (2.6) by closing the contour of integration in the inverse transformation formula

$$(u_z, \psi) = (2\pi)^{-1}\int_{i\delta-\infty}^{i\delta+\infty}(u_z^F, \psi^F)^- e^{i\omega t}d\omega, \quad \delta > 0$$

by a semicircle of large radius in the lower half-plane of the complex ω-plane and application of residue theory results in exactly (2.5). However, simple formulas can be obtained for u_z and ψ in any finite time interval by expanding Δ^{-1} in a power series in $e^{i\omega T}$ and term-by-term integration. Without going into details, elucidated in detail in /6/ for similar problems, we present the solution that holds in the time interval $0 < t < 2T$

$$\begin{gathered} u_z = -d_{33}(1-k_{33}^2)V_0[U(t+zc^{-1}-T) - U(t-zc^{-1}-T)] \\ \psi = V_0 zh^{-1}H(t) - k_{33}^2 V_0[U(t+zc^{-1}-T) - U(t-zc^{-1}-T) - zh^{-1}U(t)] \\ U(t) = (k_{33}^2)^{-1}[\exp(k_{33}^2 tT^{-1}) - 1]H(t) \end{gathered}$$

($H(t)$ is the Heaviside unit function).

The displacements are non-zero up to the time $t = T$ only in domains $|z| > h - ct$ adjoining the endfaces, while the potential ψ does not equal zero even in the unperturbed wave motion domain $|z| < h - ct$. Besides the constant unrelated component $\varphi = V_0 zh^{-1}H(t)$ it also has a time-varying addition $k_{33}^2 V_0 zh^{-1}U(t)$ due to the piezoelectric effect. Therefore, the solution for the displacements consists just of the propagating wave, while the sum over the eigenfunctions of the electric potential contains non-wave as well as wave components.

In the general case, the method of expansion in eigenmodes of vibration enables just one component of the non-wave electric field components to be extracted explicitly, the unrelated potential φ. The component due to the piezoelectric effect and that also has an infinite propagation velocity, is not contained explicitly in the sum $\Psi^{(m)}$ and is determined after solving problem (1.2)-(1.5).

In conclusion, we note that the method described enables the problem to be solved for more complex electrical boundary conditions as compared with (1.4). In place of the values of the potentials on the electrodes in considering applied problems, the magnitudes are often given for the currents through them or the characteristics of the outer electrical loops. In these cases, unknown values of the potentials are introduced into the boundary conditions (1.4) and are then determined from the equations for the currents in the outer loops or the charge conservation conditions.

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HYPERSINGULAR INTEGRALS IN PLANE PROBLEMS OF THE THEORY OF ELASTICITY*

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This paper is devoted to the solution of plane problems of the theory of elasticity by the method of discontinuous displacement using finite-part integrals (FPI). Two different integral equations (a real one and a complex one) with FPI's are obtained for the plane of a body with cracks. This opens the way for using arbitrary approximations of displacement discontinuities. The article contains integral formulae for FPI's used in the approximation of displacement discontinuities by polynomials of any order for internal elements and by special functions accounting for the asymptotic behaviour for the boundary elements. Therefore, prerequisites for increasing the accuracy of computations are created. The results of numerical experiments carried out indicate that there is a sharp increase (by two orders of magnitude) in the accuracy of the solution of the crack problem in which the integral formulae in question are used.

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